18th APPLICATIONS OF MATHEMATICS AND STATISTICS IN ECONOMICS AMSE
International Scientific Conference | Jindřichův Hradec, 2-6 September 2015

TOWARDS A SCIENCE OF EQUITY AND SUSTAINABILITY

WALENTY OSTASIEWICZ
Wrocław University of Economics, Faculty of Management, Information Systems and Finance, Department of Statistics, Komandorska Street. 118/120, 53-345 Wrocław
email: walenty.ostasiewicz@ue.wroc.pl

KATARZYNA OSTASIEWICZ
Wrocław University of Economics, Faculty of Management, Information Systems and Finance, Department of Statistics, Komandorska Street. 118/120, 53-345 Wrocław
email: katarzyna.ostasiewicz@ue.wroc.pl

Abstract
In this paper we propose a general framework for the science of equity and sustainability, which we call also ecosocionomics. It was derived logically from a few basic prerequisites. The first of them is that all lives are supported by an ecosystem. The Mother Earth feeds humans generously, but to certain limits of her carrying capacity, which can't be exceeded. The three goals: economic efficiency, ecosystem protection, and social fairness are not compatible, they stand rather in opposition one to another. . We share one world, therefore we have tojointly solve our common problems. This requires a general platform for policy making, which in turn requires envisioning the future. The supreme principle underlying that vision is argued to be a sustainable and equitable well-being for all kinds of beings.

***Key words**: ecological economics, sustainable development, ecosocionomics, equity.*

1. Introduction

A philosophy of the nature and of the mankind was developed and cultivatedin the ancient Greece. A man was considered as a political animal. Politics, ethics and economics were considered as three basic parts of a practical knowledge how to live. The whole philosophy was in accordance with the "natural" order. Society, as a community of communities, was embedded into the nature, and this society established various institutions for facilitating the common life. One of such institutions was an economy. After years economics was sectioned off as a separate part of all social sciences, moreover, economists managed to treat it in a privileged way. The global economy gave rise to WTO, WB, and IMF: the three institutions being responsible for the environment crises. One must agree with P. Singer who asserts that these institutions place economic considerations ahead of concerns for the environment,the animal welfare, and even the human rights(see Singer). The eternal natural order of life and things has been put on his head, as it is shown in Fig. 1.

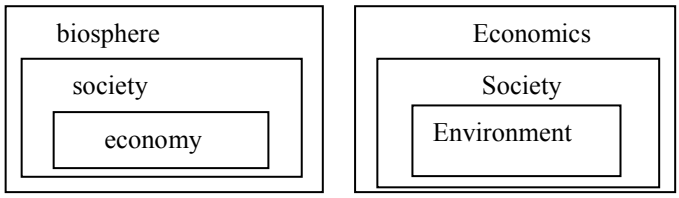

Figure 1. Natural order of things, and its reversion used by economists
Source: own drawing

2. Ethical Trilemma

Thehegemony of economists causes a lot of harmstothe environment and tothe social order. Humankind found itself at a risk of self-destruction. The first significant step towards the way out of this tragic situation has been initiated by the path-breaking report *Limits to Growth*. No less significant, but much more less popularized, is a counter-report,*Catastrophe or New Society? A Latin American World Model,* see Erikson(2010). In this model, known also as the Bariloche model, the so-called global ethical trilemma is emphasized. This means that the humankind pursues three goals: prosperity, justice, and biosphere vitality. The strange word trilemma was coined as an analog to the well known expression "moral dilemma", which refers to a situation where an agent morally ought to do each of the two acts, but cannot both. Trilemma refers to a similar situation, where one has three goals, which cannot however be pursued simultaneously. Economic efficiency is in an opposition to the ecosystem protection, but also to the social fairness. The idea of sustainability, i.e. sustainable economy, sustainable society, and sustainable ecosystem, emerged as a way to solve this trilemma. All three conflicting sets of priorities are usually depictured by means of triangle as it is shown in Fig. 2. Somewhere in the center of this triangle a place for sustainabilityis supposed to be.

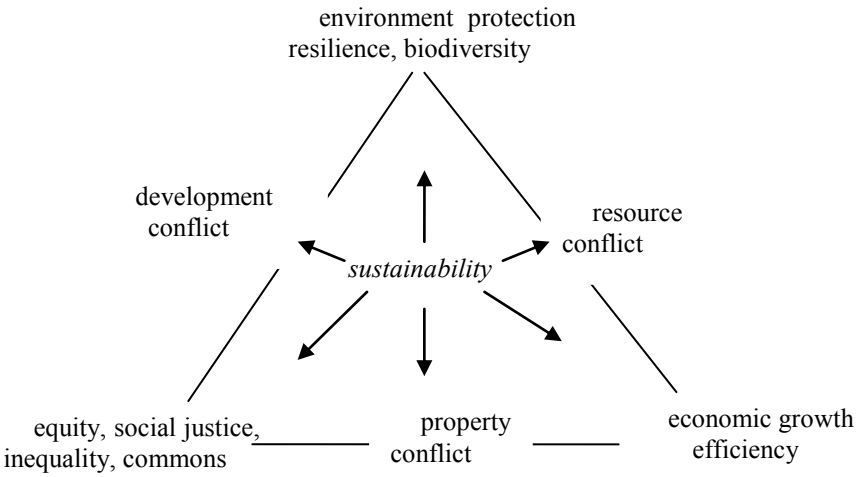

Figure 2. The triangle of conflicting goals with centrality of sustainability
Source: adaptation of Campbell (1996), Connelly(2007) and Eriksson(2010)

After the period of fragmentation of the science, we witness now an effort to unify that, what was artificially divided. The task is far from being easy. The three disciplines concerning the three domains: social system, environmental system and economy system,

represented by thecorners of the triangle shown in Fig. 2 were developed in an isolation one from another. Moreover, the goals, or the priorities, of each of these disciplines stand in opposition one to another. Kinds of conflicts are indicated on the edges of the triangle(for a deeper account of them one can consult Campbell(1996) and Connelly(2007)). How deeply these disciplines are separated, one can see from methods or models used by them. Illustrative examples of typical models used in these three domains are shortly introduced in the next paragraph.

3. Economic growth

In accordance with the ideology of the growth,the main concern in economics are growth theories. For the illustration it is enough to present a typical model of new trends of the so-called endogenous growth paradigm. The essence of this new wave lies in that, that the growth in neoclassical economics was accounted for by capital accumulation, and now the growth is caused by innovation i.e. by technological progress. This is supposed to be unlimited. For the illustration consider the economy with the three kinds of goods: a final good(Y), a constant measure of specialized intermediate products (normalized to 1), and a labor (L). The final good is produced according to the following production function (Aghion,2007):

$$Y_t = \int_0^1 A_{it} L^{1-\alpha} x_{it}^{\alpha}\, di, \quad 0 < \alpha < 1, \qquad (1)$$

wherex_{it} is the flow of intermediate input i used at t, A_{it} is a productive parameter of i, which measures the quality of the input.

It is worth to compare this function with the neoclassical production function $Y_t = L^{1-\alpha}K^{\alpha}$, and observe that now, in this new model, the output is produced by a continuum of intermediate products (capitals).

4. Inequality and poverty measurement

In all the social sciences the basic problem is inequality and poverty. As regards the measurement of these two features characterizing society, one can say that there are developed a number of satisfactory models. As good representative work can serve the article by Barrett and Salles (see Barrett(1998)), on which our survey is based. Suppose that F(y) denotes the distribution function of income Y, and that the $Y = \{(i, y_i): i \epsilon S\}$ is a distribution of income over a finite society S of size n>2.

A1. Axiom1 (invariance). If distributions of income Y_1 and Y_2 over societies S_1 and S_2 have the same distribution functions, then $I(Y_1) = I(Y_2)$.

A2. Axiom 2. I(Y) is twice continuously differentiable with respect to individual incomes.

A3. Axiom3. $I(Y_e) = 0$, with $Y_e = \{(i, \bar{y}): i \epsilon S\}$ i.e. egalitarian distribution

A4. Axiom 4. (S-independence). There exist a function h such that a transfer of δ from i to j implies the "increase"", ΔI ,of the inequality expressed by $h(n, y_i, y_{j,}\delta)$.

A5. Axiom5. (A-independence). Function $h(n, y_i, y_{j,}\delta)$ substituted by $h(n, y_i - \bar{y}, y_j - \bar{y}, \delta)$

A6. Axiom 6. (R-independence). Function $h(n, y_i, y_{j,}\delta)$ substituted by $h(n, \frac{y_i}{\bar{y}}, \frac{y_j}{\bar{y}}, \delta)$

It is proved (Barrett,1998) that if $I = I(Y)$ satisfies A1, A2, and A4, there exists a continuous function $w = w(y)$ such that the increase ΔI is given by

$$\Delta I = n^{-1}\delta \int_{y_i}^{y_j} w(y)dy + o(\delta) \qquad (2)$$

where $o(\delta)$ denotes a term which ratio to δ tends to zero as $\delta \to \infty$.
The three classes (simple, absolute, and relative) of inequality indices are derived from these axioms, and they are distinguished by the last three axioms.

The *simple* inequality index satisfies Axioms 1,2,3 and 4, and is defined by the expression

$$I(Y) = n^{-1} \sum_{i \epsilon S} \int_{\bar{y}}^{y_i} w(y)\,(y_i - y)dy \quad (3)$$

Absolute inequality (satisfies A1, A2, A3, and A5), and is defined by the same formula substituting y_i by $y_i - \bar{y}$.
Relative inequality, satisfying A1, A2, A3, and A6, is defined in the same way: substituting y_i by $y_i/\bar{y}$.

If the weight function has the form $w(y) = ky^{-c}$, then relative inequality index defines the generalized entropy family of measures. Coefficient *c* captures the "inequality aversion" (for more about this notion one can find in (Ostasiewicz,2014).

The above measure of inequality is extended to a poverty measure, by introducing a poverty line L, and by replacing $w(y)$ by $w(y, L)$. The poverty measure is defined by the formula:

$$P(Y) = n^{-1} \sum_{y_i < L} \int_{L}^{y_i} w(y, L)(y_i - y)dy. \quad (4)$$

One assumes that $(y, L) > 0$, when $y < L$, and $w(y, L) = 0$, when $y \geq L$ (for details see (Barrett,1998).

5. Discrete choices with social interactions

The most popular model in this family of models is the Brock-Durlauf model. It concerns the actions undertaken by the group of individuals. Suppose that the group consists from N individuals, and their actions (decisions) are denoted by N-tuple $\omega = (\omega_1, \omega_2, \dots, \omega_N)$, with $\omega_i \epsilon \{-1,1\}$. One assumes that act or decision ω_i is a realization of a random variable A_i . With each decision its utility is associated (Brock, 2001):

$$U(\omega_i) = u_I(\omega_i) + u_G\left(\omega_i, \mu^e\,(\omega_{-i})\right) + \varepsilon(\omega_i), \quad (5)$$

where $u_I(\omega_i)$ is an individual (private) utility associated with a choice and u_G is the group utility, $\omega_{-i} = (\omega_1, \dots, \omega_{i+1}, \omega_{i-1}, \dots, \omega_N)$ denotes the choices of all decision makers other than i, $\mu^e\,(\omega_{-i})$ denotes the conditional probability measure individual i places on the choices of others at the same time, $\varepsilon(\omega_i)$ is a random term . It is assumed that

$$Prob(\varepsilon(-1) - \varepsilon(1) \leq x) = (1 + \exp(-\beta x))^{-1} \quad (6)$$

$$u_I(\omega_i) = h\omega_i + k, \quad \text{with} u_I(1) = h + k \quad , \quad \text{and} \quad u_I(-1) = k - h. \quad (7)$$

$$u_G\left(\omega_i, \mu^e\,(\omega_{-i})\right) = J\omega_i \bar{m}_i^e \quad (8)$$

$$\bar{m}_i^e = \frac{1}{N-1} \sum_{i \neq j} m_{i,j}^e. \quad (9)$$

$m_{i,j}^e$ denotes the subjective expected value from the perspective of an individual i of individual's j choice. With these assumptions one derives that $P(\omega_i) = P(A_i = \omega_i)$ is the following:

$$P(\omega_i) = \frac{\exp(\beta\,(u_I(\omega_i) + J\omega_i \bar{m}_i^e))}{\sum_{\gamma_i \epsilon \{-1,1\}} \exp(\beta\,(u_I(\gamma_i) + J\gamma_i \bar{m}_i^e))} \quad (10)$$

Adopting the hypothesis of *rational expectations*:

$$m_{i,j}^e = E(\omega_j), for\ all\ i\ and\ j \quad (11)$$

one arrives at the following expected decision:

$$E(\omega_i) = tanh(\beta h + \beta J(N-1)^{-1} \sum_{j \neq i} E(\omega_j)). \quad (12)$$

The individual choices $\bar{m}_i^e$ in (10) are equal to a common value, which is denoted as m^* and is defined implicitly by the equation

$$m^* = \tanh(\beta h + \beta J m^*). \quad (13)$$

This value is called a stationary state, or expected average choice value, or self-consistent equilibrium.

It has been proved (Brock, 2001) that there exists at least one self-consistent choice, and moreover, that when the size of the group tends to infinity, then

$$E(\omega_i) = \bar{m} \quad (14)$$

In (Ostasiewicz, 2011) the Brock-Durlauf model has been significantly extended into two directions. First, there has been examined the dynamical version of the Brock-Durlauf model and its stationary states(when mean choice does not change in time: $\bar{m}^t = \bar{m}^{t-1}$). Moreover, it has been suggested how to apply the Holling's resilience measures to Brock-Durlauf model.

6. Food web modeling

Ecology as a science of the biosphere, i.e. the sphere of life and where life dwells, is very rich in formal models. Formulated models are not only very fine from mathematical point of view, but contrary to economic models they are closely related to the real situations. This assertion is evidenced by an example of two models exposed below.

Model of grazing pressure, see (Brännström, 2012).

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - c\frac{X^p}{X^p + H^p} \quad (15)$$

K denotes carrying capacity, *c*-maximum grazing rate, *H*- grazing population (herbivores), *X*- nutrient, and *p* is a parameter.

The other model concerns a food web structure. It is the known as the Cropp-Gabric three-compartment food web model, see (Kristensen, 2003):

$$\frac{dP}{dt} = \mu_P P\left(\frac{N}{N+k_P}\right) - e_Z PZ \quad (16\ a)$$

$$\frac{dZ}{dt} = e_Z(1-\eta_Z)PZ - d_Z Z \quad (16\ b)$$

$$\frac{dN}{dt} = d_Z Z + e_Z \eta_Z PZ - \mu_P P\left(\frac{N}{N+k_P}\right) \quad (16\ c)$$

P, Z, and N are Phytoplankton, Zooplankton, and Nutrient, correspondingly

e_Z is the consumption per day of phytoplankton mass per zooplankton mass

μ_P is the maximum phytoplankton nutrient uptake rate

k_P is the nutrient half saturation concentration for phytoplankton

d_Z is the zooplankton mortality

η_Z is the efficiency of zooplankton conversion of nutrient into biomass.

An excellent review of the models used in the ecology is given in (Brännström, 2012).

7. Towardsthe consilience

At least officially, since 1987 we started to think about our common future. Our common future depends only on us, on our common efforts, and our common will to save the planet for our grandchildren. Fortunately, there is no necessity for invention any theoretical base for this joint endeavor. The base has been already founded by ancient Greeks who discovered that we abide in the Cosmos. This Cosmos, according to Plato, has come into existence as a Living Creature endowed with soul and reason. From this follows that our common future should be devised with regard of the future of all inhabitants for whom the Earth is an abode as well,

as for human kind. The two notions frequently used nowadays: economics and ecology have the same root in the Greek word οικος, which means house, or institution. The word *economia*was used by Aristotle to define a knowledge on household management. A knowledge about all things necessary for providing well-being for all living in the house (ikos). The word *ecology*, although introduced much later, means also a house, where any life is dwelling, a single-celled algae as well as very complex, multi-celled *homo sapiens*. Economy, in Aristoteliansense, was very attractiveas a science. The word "economy" and the terminology from this science were extended into two directions, to the macrocosm as well as to the microcosm. This means that the whole Nature was considered as οικος; on the other hand, the human body was also conceived as a οικος. This was the reason for publishing works such as *OeconomiaNaturae,OeconomiaAnimalis*, economy of plant, or even economy of a women's body. Pre-classical, and afterwards classical economy were developed within a teleological framework, aimed at the good life. Paradoxically, in XX century economics broke away from a real background, and is cultivated now more as a mathematics than a science concerning a real life. On the other hand, ecology is becoming now the most comprehensive and diverse amongst sciences. Economics ought to follow now the example of ecology. The three huge real problems: social ailment, economic polarization, and environment degradation, were caused by an ideology based on profit and economic prosperity, as well as by scientific tradition based on reductionist paradigm, which is unable to address the complex dynamic problems of the living systems. After about three centuries of a mechanistic (clockwork) world view, the ancient image of the nature as an organism is revived. Moreover, according to Bergson and Whitehead it is even amplified. Following this philosophical track there are undertaken efforts to create a new science, which should have rather holistic nature than be fragmented. The firm ground for this new science should be criterion which was for the first time soundly declared in the Bariloche model, and it means that we need, above all,to create the society intrinsically compatible with its environment. The solution to the trilemma could not have the form "pick two -ignore the third" (Eriksson, 2001), but it consists in development a new framework of a comprehensive science withinwhich all problems would be considered simultaneously. Beforecreating or developing such a science one needs to have a clearvision of it. Envisioning science of sustainability is nowadays very hot topic. Different authors appealed for the usage of graphical representations which might help the understanding. Following this suggestion, we also propose an image in Fig. 3 with an intention to raise the awareness of a necessity to look from above at the various problems referring different sphere of human activities. Aristotle wrote that the entirety is something over and above its parts, and not just the sum of them. In the case considered here, these "parts" constitute three domains, which were used to be called three pillars of sustainable development.The three vertices of the base of the tetrahedron shownin Fig. 3 represent these three domains: Biosphere (ecosystem), Economy, and Society, and the appropriate academic disciplines. The edges of this triangle symbolize the disciplines which intend to link two adjacent domains: ecology and economics, society and economics, society and ecology.

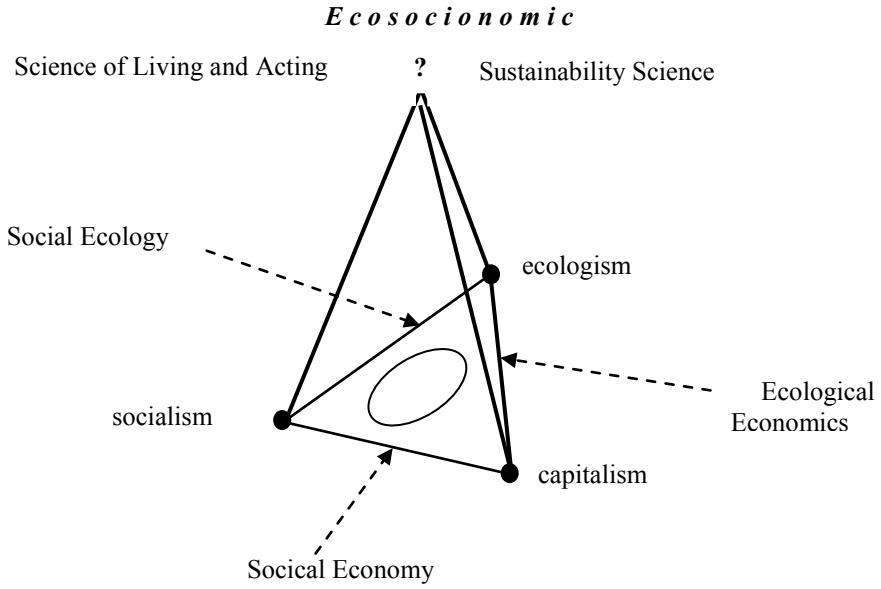

Figure 3. Tetrahedron of ecosocionomics
Source: own drawing

The vertex of the tetrahedron is intend to represent a consilience of all the knowledge necessary to develop linkages between the disciplines concerning all the domains represented by the plane of the triangle. It is quite consensual requirement to build a new science on a solid ethical foundations. Looking for the most universal principle, one might recall Schweitzer's famous saying: I am life that wants to live, in the midst of life that wants to live (in original: *Ich bin Leben, das leben will, inmmitten von Leben, das leben will*). This general principle could be supplemented by the particular commandments, like those proposed by Georgescu-Roegen(Gowdy, 1998): Thou shalt love thy species as thyself, Thou shalt minimize regrets instead of maximizing thy utility. It seems also to be indisputable ethical principle that people should not be treated differently because of their geographical location, belonging to different states, or even of living in the future. Confronting this indisputable principle with Bush's announcement: "We will not do anything that harms our economy, because first things first are the people who live in America", R. Prodi responded him highly eloquently "if one wants to be a world leader, one must know how to look after the entire Earth and not only American industry". How sad condition of our civilization is when one has to be in an awful dilemma: "Americans or ethics?" (Singer, 2004).

8.Conclusions

Researches in the three domains: natural environment, society, and economy are briefly reviewed. The scope of this review was to highlight the basic problems and differences in the approaches used to solved them. It was argued that the problems concerning each of these three domains can't be solved by the techniques developed inside these domains. This assertion coincides with the famous Einstein's dictum: we cannot solve our problems with the

same thinking we used when we created them. Therefore we propose to develop a sort of consilience science in which the techniques will be constructed outside of the domains. Using metaphor, we assert that the problems on the plane should be considered from the above, in order to see simultaneously problems belonging to the three different fields.

References

1. AGHION P., HOWITT P. 2007. Capital, innovation, and growth accounting, Oxford Review of Economic Policy, vol. 23, pp. 79-93.
2. BARRETT R., SALLES M. 1998. On three classes of differentiable inequality measures, International Economic Review, vol. 39, pp. 611-621.
3. BRÄNNSTRÖMA., et al. 2012. Modelling the ecology and evolution of communities: a review of past achievements, current efforts, and future promises, In Evolutionary Ecology Research, 14, pp. 601-625.
4. BROCKW. A., DURLAUF, S. N.2001. Discrete choice with social interactions, Review of Economic Studies, 68, pp. 235-260.
5. CAMPBELL, S.1996. Green cities, growing cities, just cities? Urban planning and the contradictions of sustainable development, In Journal of the American Planning Association, vol. 62, iss. 3, pp. 296-312.
6. CONNELLY S.2007. Mapping sustainable development as a contested concept, Local Environment, vol. 12, iss. 3, pp. 259-278.
7. COSTANZA R. 2003. A vision of the future of science: reinterpreting the study of humans and the rest of nature, "Futures", 35, pp. 651-671.
8. ERIKSON R., ANDERSSON J. O. 2010. Elements of ecological economics, Routledge.
9. GOWDY J., MESNER, S.,1998. The evolution of Georgescu-Roegen'sbioeconomics, In Review of Social Ecology, 2, pp. 137-156.
10. KRISTENSEN N. P., et al.2003. Is maximizing resilience compatible with established ecological goal function?. Ecological Modelling, 169, pp. 61-71.
11. LELE, S. 1991. Sustainable development: a critical review, World Development, vol. 19, iss. 6, pp. 607-621.
12. MITMAN G. 2006. Where Ecology, Nature, and Politics meet: Reclaiming *The Death of Nature*, Isis, 97, 2006, pp. 496-504.
13. OSTASIEWICZ, K., 2014. Impact of outliers on inequality measures- a comparison between Polish voivodships, In Silesian Statistical Review, vol. 12, iss. 18, pp. 105-120.
14. OSTASIEWICZ, K., MAGNUSZEWSKI P. 2011. The concept of resilience and its application to a certain socio-econimical model, PragmataTesOikonomias, iss. 4, pp. 203-216
15. SINGER, P. 2004. One world, the ethics of globalization, Nota Bene, Yale.
16. WILSON, E. O. 1998. Consilience: the unity of knowledge. Knopf, New York.