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EUROPEAN CALL OPTIONS FOR THE FREIGHT RATE IN CONTAINER SHIPPING

ALBERT GARDOŃ

Wroclaw University of Economics, Institute of Applied Mathematics, Chair of Statistics,
Komandorska 118/120, PL-53-345 Wrocław, POLAND
email: Albert.Gardon@ue.wroc.pl

Abstract

Models assuming the normality of real data are widely used in many disciplines but in last years this approach has been criticised because of heavy tails of empirical data distributions which contradict the normality. In order to overcome this problem models based on the so-called jump-diffusion processes are created if only the empirical data distributions seem to be normal at least after an extraction of jumps. The case of the freight rate in container shipping fulfils this requirement.

In this paper we consider the idea of the European Call options for the freight rate. Different approaches for the pricing are presented and the option premium is calculated numerically based on the jump-diffusion model. Results are calculated using the real data from one of the market leading companies and from the s.c. Shanghai Indeces. Besides, the sensibility and the impact of the European Call options in container shipping industry is discussed.

Key words: *pricing and yield (revenue) management, liner shipping, data normality, jump-diffusion, freight rate call options.*

1. Preliminaries

For the container shipping market the Herfindahl-Hirschman Index, which is a measure of industry concentration, remains below 7% (value between 1% and 15% indicates an unconcentrated industry) and the top 10 carriers have together only about 60% of the market share, see (Alphaliner, 2012). That means the market is rather competitive. In such markets the psychology of many independent competitors plays a crucial role and hence, random models become adequate for describing the behavior of the market.

Models assuming the normality of real data are widely used in many disciplines, first and foremost in finances, where the price processes are represented by the geometrical Brownian motion (the famous Black-Scholes model — see Karatzas, Shreve, 1998). Unfortunately, in last years this purely gaussian approach has been criticized because of heavy tails of empirical data distributions which contradict the normality, see (Andersen et al., 2007), (Cont and Tankov, 2004), (Das, 2002), (Johannes, 2004). However, the researchers have tried to overcome this problem by creating models based on the general jump-diffusion processes where large price changes have been extracted from the empirical distribution and modeled separately as jumps driven by the Poisson process. Differential equations describing such models must be usually solved by means of numerical methods, see (Gardoń, 2004, 2006). But it helps neither because the empirical distributions of a financial data, even after extraction of jumps, are asymmetrical or too slender, that means they have got significantly different skewness or higher kurtosis than in the normal case, see (Gardoń, 2011), (Peiró, 1999).

In the case of the container shipping industry the problem appears as well but fortunately there are parts of this market where the jump-diffusion model may be relevant, see (Gardoń, 2014). This model is defined by the following stochastic differential equation in the integral form:

$$X_t = X_{t_0} + \int_{t_0}^{t} aX_s ds + \int_{t_0^+}^{t} b\overline{X}_s dW_s + \int_{t_0^+}^{t} \overline{C}_s \overline{X}_s dN_s , \qquad t > t_0 \quad (1)$$

where the modeled process X denotes the weekly average net freight (or rate, which is a transportation price consisting of basic ocean freight and different surcharges like e.g. fuel surcharge) per transported unit (TEU or FFE — the volume of a 20 or 40 feet long standard container), $\overline{X}_t = X_{t^-} = \lim_{s \to t} X_s$, W is a standardized Wiener process, N is a homogeneous Poisson process with intensity λ and the both driving processes are said to be independent. Coefficients a, b and C (overlined C_s means the left-hand side limit at s, as for X) are called here the drift, the volatility and the relative jump size, respectively. Further, C denotes a strictly stationary process independent from the both driving processes W and N, see (Mancini, 2009). This means the jump sizes are realizations of an independent identically distributed (iid) sequence of random variables (C_{Γ_i}) which may be treated as independent copies of a certain random variable C, whereas the jump times of the process N are denoted by (Γ_i). Additionally, the sequence (τ_n) will represent times when the process is observed.

2. Derivatives

In this paper we would like to introduce the idea of the derivatives into the container shipping market on the simplest example of the European Call options, see (Karatzas and Shreve, 1998). The idea is strongly considered in recent years. In (Koekebakker et al., 2007), asian-style options were priced based on the continuous diffusive model. In (Nomikos et al., 2013), the authors considered aspects of asian options pricing based including jumps in the model. European options are a basic example of derivatives. They give the owner the right for buying (Call) or selling (Put) an underlying good at the fixed time (expiration date) in the future for a fixed price (strike price). This underlying good is usually an asset or a market index but it may be also a physical commodity as e.g. grain. In our case it would be a shipping service.

Another type of derivatives is already widely used in the shipping industry – forward/future contracts. They are rather informal and consist in long term agreements between shippers (commodity owners) and carriers (vessel owners). Unfortunately, such contracts are unreliable and therefore cannot be an effective hedging tool for the risk reduction. If the average market freight rate changes significantly, especially if it drops, the shippers strongly insist the carriers for renegotiation and reducing the previously contracted prices. Since the market is competitive the carriers have no other choice as to decline rates in order not to lose customers. In addition, the freight rate volatility is rather large. In the crutial part of the market, transportation route from South-East Asia to Europe, the freight rates have varied in recent years between 2000 and 4000 USD/FFE. Thus, there is a great need among the shippers for a tool that would minimize the income fluctuations. It might be delivered by freight options.

The main difference between a future contract and an option is that the first obliges both contractors to realize the underlying deal whereas the latter let the owner make the decision wether the contract is realized. For instance, if one company has a future contract for buying a certain commodity for 1000 USD and another company has an European Call option for the same commodity with the strike price 1000 USD, both with the expiration date a month ahead, then if a month later this commodity will cost 900 USD the first company will have to realize the future contract and to pay for the commodity 1000 USD whereas the second one may not realize the Call and buy this commodity for 900 USD. Since an option is a special right for its owner it cannot be for free. The cost of an option is called the premium and is calculated as the discounted expected value of the future profit with respect to the martingale probability measure, see (Karatzas and Shreve, 1998), (Nomikos at al., 2013). We will present it in next sections how the fair premium is calculated.

The idea of European Call options should resolve the problem of price renegotiations in the case of freight drop. The premium is paid by an option buyer at the time of the enter into the agreement and is usually a tiny part of the strike price. In the instance of a freight decrease in the future a shipper which bought the Call will have the possibility for shipping the commodities for the market price lower than the strike price and resigning from the option. A carrier will earn in this case less for the transportation but it already received an addional income for issuing the Call compensating the freight rate drop. On the other hand, if the rates will increase, a shipper will have the guarantee that the cargo will be shipped for a previously agreed lower strike price. A carrier will gain in this instance less than it could regarding actual market rates, though, it will still receive the satisfactory strike price from the option plus the premium already gained when the Call was issued. This operation should reduce the freight change risk for both contractors, hence, should be advantageous for both parties.

At the end of this section we want additionally to explain why Put options are not a natural derivative in this industry. A Put option gives the buyer the right for selling in the future a good or a service at the strike price. However, a natural seller is of course a carrier offering the shipping possibility whereas a natural buyer is a shipper which wants to use the service and this relationship cannot be reversed. Hence, a carrier cannot offer that it will buy any good or service from a shipper in the future. Nevertheless, Put options might be led into the container shipping market but in a virtual sense. This solution is used on financial markets where an underlying asset is not sold physically. An owner of a Put does not sell anything just earns the difference between the strike price and the actual market price of an underlying asset if the difference is positive. But such an approach requires strict legal regulations like those for stock exchange markets.

3. Option pricing

As it was mentioned in the previous section the net premium of a European Call option is generally calculated as the discounted expected value of the future profit with respect to the objective (risk free) martingale probability measure Q, i.e.:

$$O_t = E_t^Q\left(e^{-\rho(T-t)}\max(X_t - K, 0)\right), \qquad t > t_0 \quad , \tag{2}$$

where X is the net freight process, T is the expiration time, K is the strike price, ρ is the riskless rate and LIBOR is usually taken as such a rate. Q is called a pricing probability measure such that the net freight process X is a local martingale with respect to Q. The measure Q realizes the so-called no arbitrage assumption, see (Karatzas and Shreve, 1998) which sais that there cannot be an opportunity on the market for a riskless relative profit greater than ρ. In practice, this assumption requires that in Eq. (1) the drift coefficient $a = \rho - \lambda EC$, see (Kou, 2002).

If in the Eq. (1) the jump component vanishes, i.e. if C=0 a.s., then the jump-diffusion equation reduces to the well known Black-Scholes model, see (Karatzas and Shreve, 1998) which is widely used for pricing the derivatives on financial markets. In this instance the explicit formula for the Call net premium is known:

$$O_t = \Phi(d_+)X_t - \Phi(d_-)Ke^{\rho(T-t)}, \tag{3}$$

where

$$d_{\pm} = \frac{\log\frac{X_t}{K} + \left(\rho \pm \frac{b^2}{2}\right)(T-t)}{b\sqrt{T-t}}$$

and Φ is the standard normal distribution function. Since this purely gaussian approach has been criticized in last years because of heavy tails of empirical distributions in our article we will compare the premium evaluation (3) to the premium calculated based on the jump-diffusion model (1). In this latter case it cannot be calculated explicitly and it will be simulated by means of the Monte Carlo method. Hence, the jumps have to be identified and the model needs to be calibrated.

We decided to conduct the calculations based on the real data from one of the leading carriers. The data set consists of 656 weekly net freight returns from the over 12 years long time period from January 2 2000 to August 5 2012 concerning the most important and competitive South-East Asia – Europe trade, head haul (more profit-yielding) direction (see Fig. 1). The data seems to be normally distributed apart from a set of outliers and the jump-diffusion model seems to be relevant in this instance, see (Gardoń, 2014).

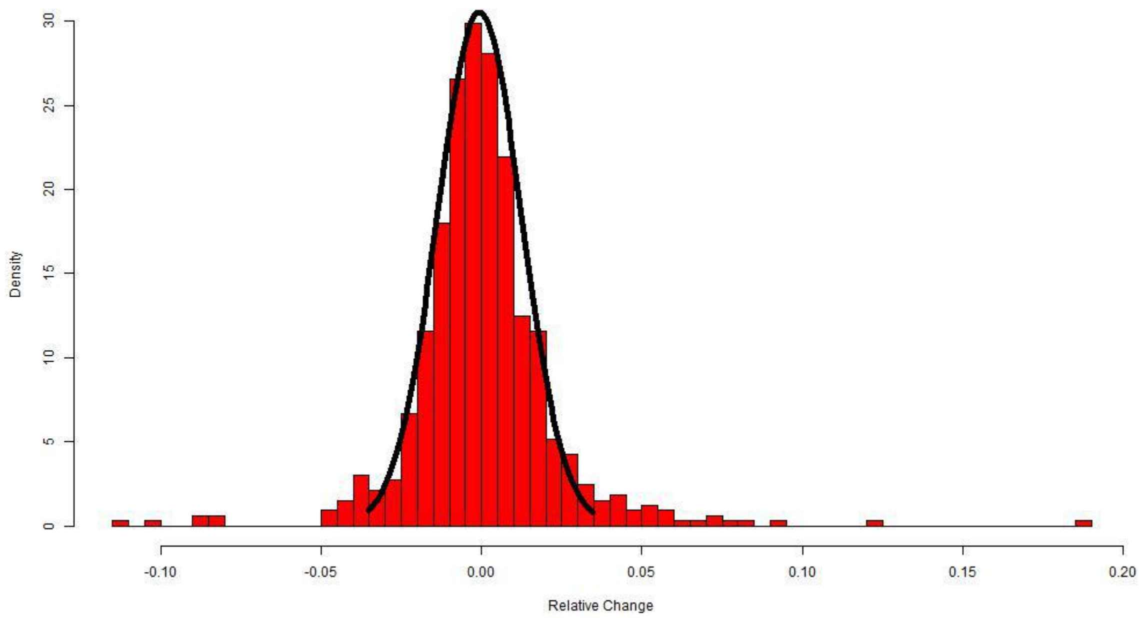

Figure 1. The normal vs empirical density of the relative rate changes for the Asia–Europe trade.

The Black-Scholes model requires that the discretized relative price changes:

$$Z_n = \frac{X_{\tau_n} - X_{\tau_{n-1}}}{X_{\tau_{n-1}}} = \frac{\Delta X_{\tau_n}}{X_{\tau_{n-1}}} \approx \ln \frac{X_{\tau_n}}{X_{\tau_{n-1}}}, \qquad n = 1,\ldots,L,$$

where L is the number of observations, are realizations of a normally distributed random variables sequence. But since (τ_n) is not said to be equidistant the data must be firstly standardized:

$$\forall n = 1,\ldots,L \qquad Z_n^* = \frac{Z_n - a(\tau_n - \tau_{n-1})}{b\sqrt{\tau_n - \tau_{n-1}}} \overset{A}{\approx} N(0,1).$$

The symbol $\overset{A}{\approx}$ means the assymptotical distribution. Hence, the drift and volatility parameters a and b must be firstly estimated. However, as it is already mentioned the so-called no arbitrage insists on taking $a = \rho - \lambda E\mathbf{C}$. On November 19 2012 corresponding to the data set this riskless rate was equal to 0.86% p.a. for 1 year USD contracts which implies for the time unit 1 week: $\rho = 0{,}0165\%$. On the contrary, the volatility may be estimated in the maximal likelihood sense by the standard deviation of normalized returns:

$$\hat{b} = \sqrt{\frac{1}{L}\sum_{n=1}^{L}\frac{Z_n^2}{\Delta\tau_n} - \left(\frac{1}{L}\sum_{n=1}^{L}\frac{Z_n}{\sqrt{\Delta\tau_n}}\right)^2}, \qquad \textbf{(4)}$$

which follows immediately from the fact, that the relative price changes (Z_n) have to be normally distributed and b is in such an instance the infinitesimal variance of the normalized return.

In the instance of the jump-diffusion model firstly the jumps have to be recognized and extracted from the remaining "continuous" data. One of possible ways is the theshold method, see (Mancini, 2009), (Gardoń, 2011) with the threshold condition:

$$\frac{Z_n^2}{\hat{b}^2} > r(\Delta\tau_n),$$

where the threshold function $r(t) = \beta t^{1-\varepsilon} = 7{,}3576 t^{0{,}9}$ and $\hat{b}$ is evaluated by means of an iterative procedure based on Eq. (4) regarding to the remaining "continuous" (still not recognized as jumps) returns at each step. Eventually, the estimation of the Poissonian intensity λ has been evaluated for $\hat{\lambda} = 0{,}08$ and the relative jump sizes have been simulated directly from the empirical distribution.

In the Fig. 2 it is shown how the net premium based on the jump-diffusion model depends on the strike price and the time to expiration, when the actual freight rate is 3000 USD/FFE. Each case was evaluated by means of 1000 artificially generated paths of the net freight process X. Even with the strike price about 10% lower than the actual net freight the net cost of the Call has been 10-15% of the actual rate depending on the time to expiration. More precisely it is presented in Fig. 3 where the premium of the 6 months ahead Call depends only on the strike price.

The results following from the Black-Scholes model are not presented in figures because this part of investigation was conducted just for comparison. We only sum up that the premium in this instance has been usually 50-100 USD lower than in the jump-diffusive case,

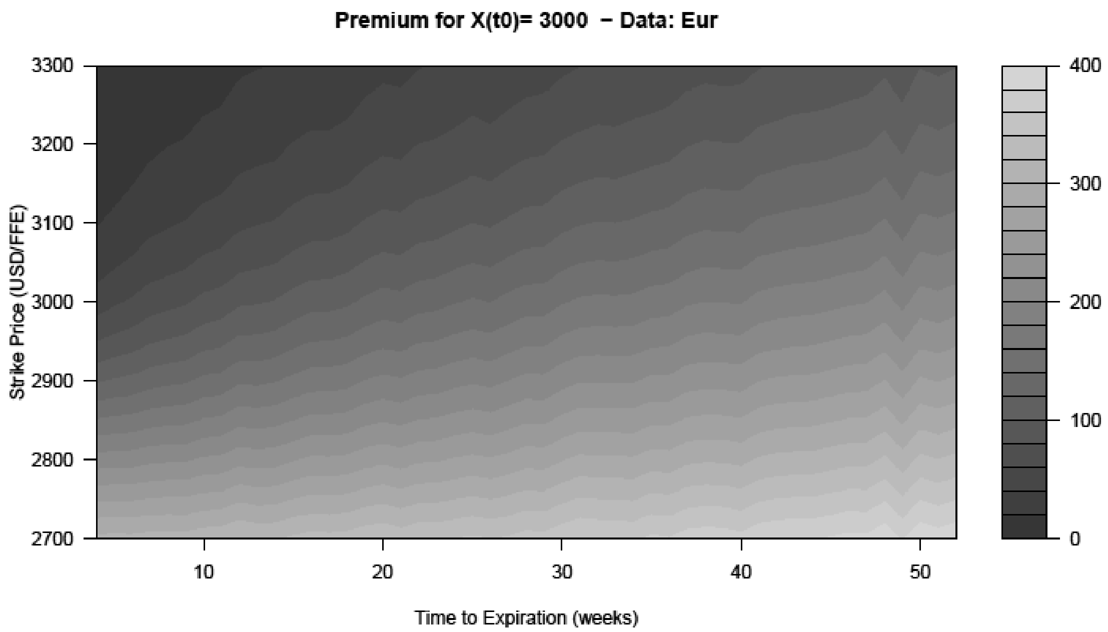

Figure 2. The net premium of the European Call for the net freight from South-East Asia to Europe with the initial freight rate 3000 USD/FFE, depending on the strike price and the time to expiration.

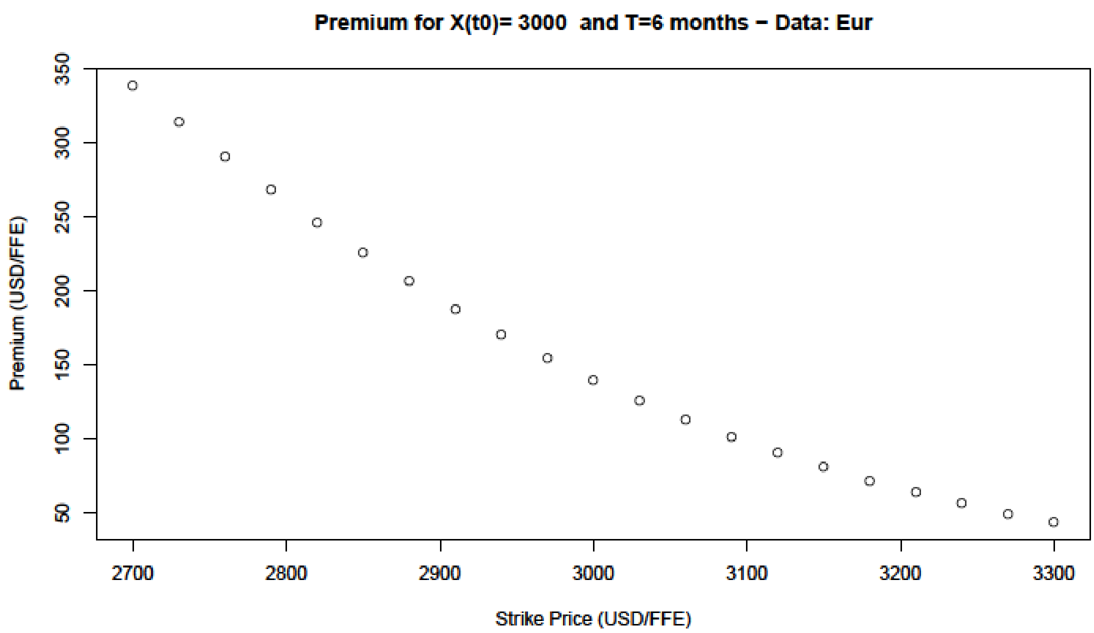

Figure 3. The net premium of the European Call for the 6 months ahead net freight from South-East Asia to Europe with the initial freight rate 3000 USD/FFE depending on the strike price.

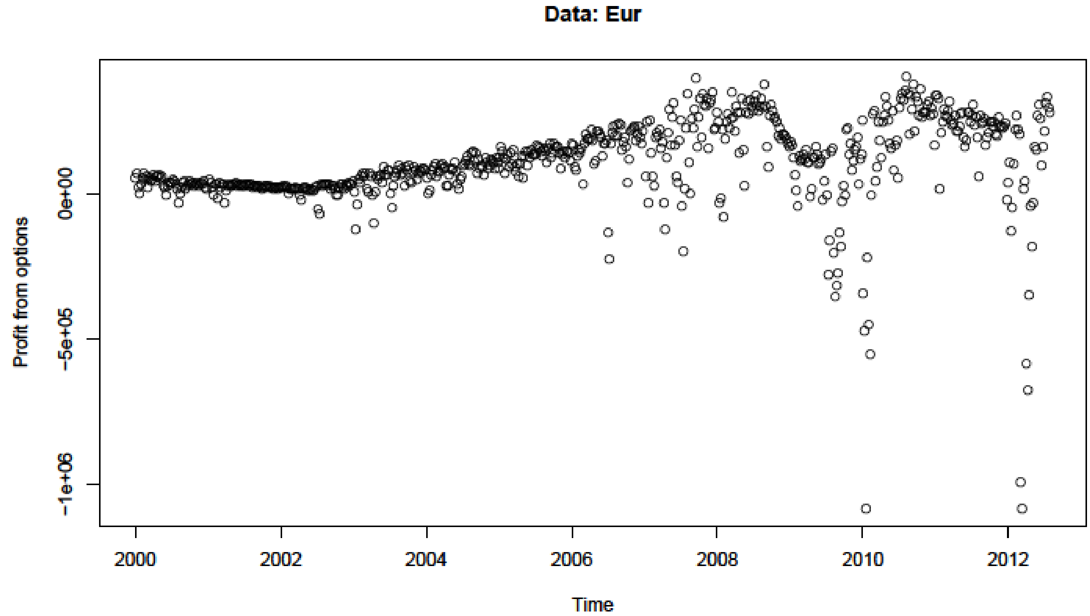

Figure 4. The simulated weekly additional company profit or loss corresponding to the European Call options issued for the net freight.

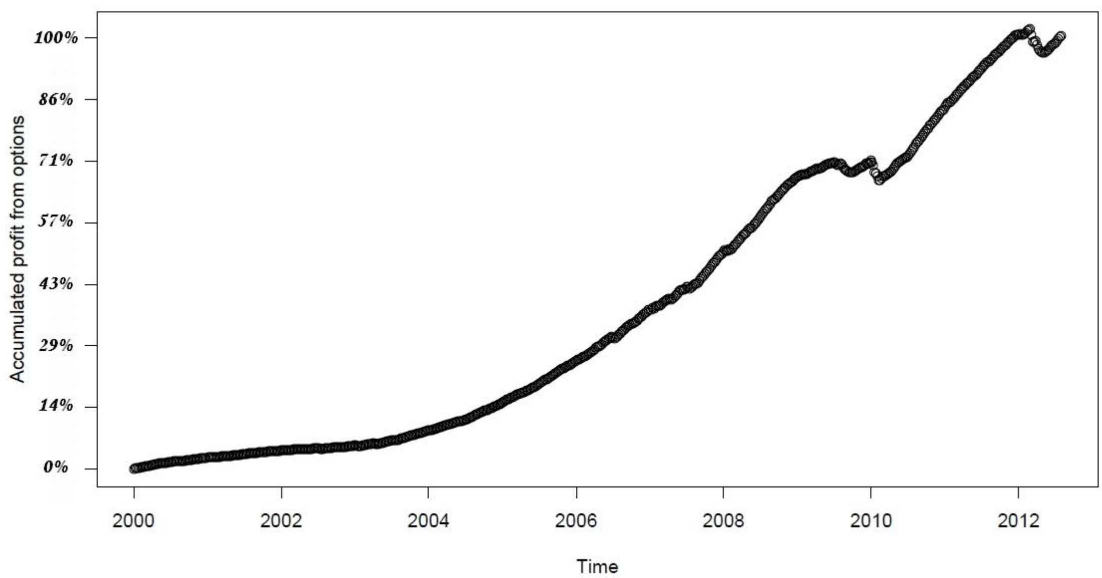

Figure 5. The simulated accumulated additional company profit corresponding to the European Call options issued for the net freight expressed by the percentage of the total profit in the yaer 2011 (the last whole year in the period investigated).

especially for strike prices strongly differing from the initial rate. This is not surprising since the gaussian models underestimate the probability for outliers.

4. Simulations

This last section we will devote for a special simulation for checking how much a company would lose or gain additionally if it issued Calls for the entire data period, i.e. from the beginning of the current millennium. The scenario consists of several assumptions:

- every week the Calls were issued for 10% of shipped containers;
- all Calls were issued for the time horizon of 3 months=91days=13 weeks;
- all Calls were issued with the strike price identical as the present net freight, i.e. K=X(t0);
- calls were issued only if the sum of the strike price and the premium exceeded the minimal profit-yielding level 2900 USD/FFE;
- to each premium the 10% commision is added but not less than 10 USD.

The level 2900 USD/FFE was arbitrary fixed, based on the previous experiences, by the company which gave us a part of its database for our disposal and we have not verified the sensibility and the exact meaning of this level. Further, we have not included the costs of the option issuing. We have believed they are negligible, especially in a longer time horizon, because the way a Call option contract would be agreed does not differ from the way the parties usually enter into a forward/future contract. The week by week additional profit or loss is drawn in Fig. 4. Already at the first look it is visible that this profit/loss behaves more stable in the first half of the time period investigated. There is a simple explanation of this fact, namely the volume of transported cargo which has been increasing systematically by 5-10% yearly for last 15 years. The increasing absolute amount of possibly issued Calls enlarged the volatility of the profit in the second half of observations.

Generally, the issuing of Calls would have brought the carrier a slight additional profit in tough years of a price decline. On the other hand it would lead to quite dramatic losses exceeding even 1 million USD weekly during good times when the freights exploded. In those weeks the cargo would have had to be shipped for a much lower option strike price than the actual market net freight. The crucial question is what is the total impact of Calls for the company profit during the entire investigated time period. The accumulated profit/loss is shown in Fig. 5 and looks extremely optimistic. It is not monotone increasing, though, it has never appeared negative. Moreover, after almost 13 years of application of the Calls it would deliver the additional profit equal to about 100% of the total actual profit in the year 2011 which has been the last whole calendar year during the period of investigation. It is equivalent to about 8% of an extra income yearly.

5. Conclusions

Although the scenario discussed in the previous section is rather conservative, it would give the carrier issuing the european Call options for the net freight a significant additional profit. What is more, this additional profit is distributed in such a way that the Call premiums generate an additional profit in the time of price decline (when the Calls are not executed by shippers) whereas in the time of price increase the Call options executed by shippers imply the lowering of the profit for a carrier. Hence, the options decrease the volatility of the weekly net freight making the income more stable and limiting the risk of a carrier. On the other hand, the Calls assure the upper bound for the transportation price paid by a shipper limiting its risk as well. Summing it up, this type of contract between a shipper and a carrier should be attractive for both parties.

The investigation is conducted based on the data from a chosen company only, though, since this part of the market is significantly competitive, the competitors' freight rates should be quite similar one to another. Thus, the results described may be useful for each company operating in this industry.

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